ALGEBRAIC MAGIC SQUARES.

There comes a time in the school year, usually during the spring term, when the mathematics teacher becomes convinced that as far as algebra is concerned, he might just as well be teaching so many "wooden Indians." Those pupils, who are not wholly in a trance, are surreptitiously fondling a baseball glove, while x's and y's pass by unheeded. The teacher's first impulse is to give every one a good shaking in a frantic attempt to close the ever-widening gap between the intellectual capacity of his pupils and the intelligibility of his subject. He realizes something must be done at once, if his class is to learn any more algebra that year.

The introduction of graph work into the first year of the mathematics course has done much to solve this problem. A baseball graph in red and yellow colors showing the standing of the local team and the leader in its league, helps to convince the boys that there is something human about their algebra teacher, while a discussion as to whether a regulation baseball diamond is a perfect square, to which every boy enthusiastically brings the latest baseball guide, and the teacher the Pythagorean theorem with its horrible name artfully concealed, really persuades the boys that mathematics after all may have a vital bearing on the big interests of life when no amount of engineering problems could possibly do so.

For the writer, the particular bugbear of the school year is the unhappy meeting of his large classes of boys, and the two weeks just before the spring vacation. That the boys have "gone stale" is evidenced by their apathy, and more alarmingly by their restlessness. The graph work may be used to advantage to tide over this period, but to the pupil returning from his vacation, it has become an old story. Much better results have been secured by postponing this work until the boys have had a chance to play a little baseball, and by introducing magic squares into this pre-vacation period.

As their name implies, these squares were believed to have magic properties in the early days of history by various peoples

who interested themselves in numerical studies. In fact a magic square on a small metal plate hung about the neck was considered a powerful means of warding off sickness. Fig. 1 is an example of the simplest kind of magic square.

17	24	1	8	15
23	5	7	14	16
4	6	13	20	22
10	12	19	21	3
11	18	25	2	9

FIG. 1.

The middle cell of the top line is occupied by the first number, and the successive numbers are placed in their natural order as follows: when a number is placed in the top line, the next number is written in the bottom line in the nearest column at the right. Whenever it is possible, the numbers are sloped upward to the right. When a number is placed in the column at the extreme right, the next number is placed in the column at the extreme left in the line above. When none of the preceding rules can be followed, place the next number in the cell immediately below, and go on as before. When the square is completed, the same sum will be obtained by adding the numbers in the columns, lines and even the diagonals. In the example given above the sum is 65.

Why such a square should excite more than passing interest in first-year pupils, the writer will not attempt to say, but the fact is that the sight of the teacher sprinkling a few numbers here and a few numbers there in this gridiron arrangement, and then getting the same sum from the vertical, horizontal, and diagonal addition, is electrical in its effects on all the pupils and particularly on those whose mental numbness is strongly pronounced. Once their interest is caught, the pupils are encouraged to find all they can about magic squares in

encyclopedias and the books on mathematical recreations in the public libraries. As the above rules are for magic squares of an odd number of lines and columns, they are not satisfied until they have found a set of rules for the more difficult even number of cells. The work is as perennially interesting for the teacher as for the pupils, as each year some

238	143	234	153	228	241	147	231	154	146
148	214	161	221	171	166	211	173	215	235
224	165	204	181	209	177	175	203	218	159
144	220	183	191	186	197	192	200	163	239
158	219	205	193	196	187	190	178	164	225
232	160	176	184	189	194	199	207	223	151
227	170	210	198	195	188	185	182	213	156
157	216	180	202	174	206	208	179	167	226
150	168	222	162	212	217	172	210	169	233
237	240	149	230	155	142	236	152	229	145

FIG. 2.

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unusually ingenious form of magic square, like Fig. 2, is unearthed by some one in the class from juvenile magazines at home. This example illustrates what is known as a bordered magic square, as the borders may be successively removed without destroying its "magic" properties until it is reduced to four columns and four lines. The numbers in each of the inscribed crosses have the same total as a line or column of the complete square.

Only one period of class time is necessary to give this work a good start, and for the rest of the time it may be carried on along with the regular course. The question naturally arises as to whether the fascination of this work does not cause the pupils to neglect their algebra. Such is not the case. The interest aroused by the magic squares secures a mental alertness which transfers to the other work. Furthermore, it is an interesting and very significant fact that most of the pupils are sure to become restless and even a little worried if the regular algebra work is wholly put aside for any length of time.

A happy combination is secured by assigning a magic square in addition to the usual home-lesson; first a square of 25 cells beginning with the number 1, then a larger number of cells and larger initial numbers. The addition of every line, column and diagonal must be insisted on as the only true test of a genuine magic square. Quick methods of addition can be very profitably introduced here, and even a passing mention of lightning calculators and their feats will do much to heighten the growing interest.

Soon the boys are engaged in a contest to see who can make the largest magic square. As a rule they try to secure extra credit for this work which may very properly be allowed. At times the writer has had misgivings as to whether magic squares with a hundred cells on a side and drawn on a piece of wrapping paper two or three feet square did not signify an interest which had been carried to an unreasonable extreme. However, as there are no injurious reactions, and as the boys seem to get so much fun out of it, a sympathetic attitude may well be shown by the teacher so long as the pupils are willing to do the necessary addition.

When they seem reluctant to do this, the time has come to turn to algebraic squares. Indeed, sooner or later, some one is sure to ask if there is such a thing as an algebraic square. A search in the library fails to reveal any description of one, so the pupils are invited to make them up themselves. The first attempt is fairly sure to be the usual magic square with some one letter placed after each number. Then they are asked to make a square in which both plus and minus signs are used. There is no rule given them, but they must make it as

best they can by guessing and trying. Squares whose lines and columns have the same sum are accepted for credit, even if the diagonals do not. This rule places the task within the

$8x+5$	$x-9$	$6x+1$
$3x-5$	$5x-1$	$7x+3$
$4x-3$	$9x+7$	$2x-7$

FIG. 3.

mental reach of every member of the class, and it is really touching to see the time and energy that the lowest tenth of the class will put in on this work. It is hardly necessary to say that the formation of a square of even nine cells of this

$4x$	-3	x^2	-1	$2x$
-1	$2x$	$4x$	-3	x^2
-3	x^2	-1	$2x$	$4x$
$2x$	$4x$	-3	x^2	-1
x^2	-1	$2x$	$4x$	-3

FIG. 4.

kind gives considerable practice in algebraic addition. From monomial cells the next step is the binomial cells which are not much more difficult. Fig. 3 is an example of this kind of square. Polynomials may also be attempted, but are rather unwieldy.

Multiplication magic squares are next in order; that is, squares whose lines, columns and diagonals have the same products, as in Figure 4.

More complicated magic squares like Figs. 5 and 6 are encouraged and accepted for extra credit, but are not required of the whole class. All of these examples in algebraic magic

$18x^2-20x \div 2x$	$16x^2-24x \div 8x$	$21x^2-24x \div 3x$
$16x^2-20x \div 4x$	$12x^2-14x \div 2x$	$40x^2-45x \div 5x$
$15x^2-18x \div 3x$	$20x^2-22x \div 2x$	$18x^2-24x \div 6x$

FIG. 5.

squares are the work of Boston English High School boys. The sums of the products or quotients in each of these figures are the same.

$2(x-4)$	$3(2x+3)$	$5(4x-1)$	$7(8x-9)$	$8(6x+2)$
$7(8x-9)$	$8(6x+2)$	$2(x-4)$	$3(2x+3)$	$5(4x-1)$
$3(2x+3)$	$5(4x-1)$	$7(8x-9)$	$8(6x+2)$	$2(x-4)$
$8(6x+2)$	$2(x-4)$	$3(2x+3)$	$5(4x-1)$	$7(8x-9)$
$5(4x+1)$	$7(8x-9)$	$8(6x+2)$	$2(x-4)$	$3(2x+3)$

FIG. 6.

The interest of those pupils who are unable to make the more difficult squares can be kept up by requesting them to check the accuracy of the magic squares when they are put on

the board. This can be done quickly by requiring the first row of the class to find the sum of the top line of products or quotients, the second row the second line, etc. This work is eagerly pursued by most of the class, as most of them cannot be convinced that a task, in which they themselves have failed, can possibly be performed until every line, column and diagonal has been checked up.

The following simple rule for a square of five lines and columns is finally discovered by the pupils. The top line is made up of cells containing indicated multiplications or divisions. This line of cells is then, repeated in the next four lines in the order shown in Fig. 7. The fourth cell in the first line becomes the first in the second line and so on.

1	2	3	4	5
4	5	1	2	3
2	3	4	5	1
5	1	2	3	4
3	4	5	1	2

Fig. 7.

Finally when the last algebra period on the last day of school before the spring vacation arrives, the teacher rubs his eyes to find even some of his mathematical "black sheep" still lingering to ask questions about magic squares, and he becomes convinced that while some may be taught by a direct attack, others must be taught by stealth.

Henry P. McLaughlin.

English High School,
Boston.